Numeracy Literature Review for Evidence Based Practices Framework
The Smarter Schools National Partnership on Literacy and Numeracy is a joint initiative of the Australian Government and the Department of Education and Training, Northern Territory Government.

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Evidence Based Literacy and Numeracy Framework

Literature Review: Numeracy

Purpose of the Literature Review

This literature review aims to distil the critical features of effective numeracy classroom practices from a wide range of current studies and research to inform the design of an Evidence Based Numeracy Practices Framework. The purpose of the Framework is to define and explain the repertoire of practices that research has found will lead to improved numeracy outcomes for students. It is proposed to use this Framework to inform the evaluation and/or development of numeracy programs for schools in the Northern Territory (NT).

Scope

It is important to define the research parameters for this literature review. Firstly, it has focused on recent, major studies into effective numeracy teaching and learning practices conducted in the United Kingdom (UK), United States of America (USA), New Zealand (NZ) and Australia. Secondly, for the purposes of developing an evidence based literacy and numeracy framework that could be used at the system level, it was decided to focus on research that involved a meta-analysis of current research or large-scale studies conducted in a wide range of schools. Findings from these studies are more likely to be transferable when applied to the range of learning contexts in the NT. Some smaller scale studies have been included because they elaborate on an important aspect of teaching practice that has been raised in larger scale studies or because they are of particular relevance to the NT context. Finally, the research selected for this literature review focused on effective numeracy teaching and learning for students in the early, primary and middle years of schooling (students from or just prior to school entry to years 9 or 10).

Recent research on numeracy has explored a number of discrete areas:

- The features of effective numeracy classroom teaching and learning including:
  - the content of numeracy programs
  - teaching, learning and assessment strategies.
- Characteristics of numeracy teacher effectiveness, including knowledge, beliefs and skills and participation in professional learning.
Studies in these areas have been considered as relevant to this review and have been included in this literature review.

**Learning content for numeracy programs**

**Numeracy**

Numeracy is the capacity, confidence and disposition to use mathematics to meet the demands of learning, school, home, work, community and civic life. This perspective on numeracy emphasises the key role of applications and utility in learning the discipline of mathematics, and illustrates the way that mathematics contributes to the study of other disciplines (ACARA, formerly the National Curriculum Board, 2009, p.5).

**Numeracy and mathematics**

The Australian Association of Mathematics Teachers (AAMT) has stated that:

Numeracy is not a synonym for mathematics, but the two are clearly interrelated. All numeracy is underpinned by some mathematics; hence school mathematics has an important role in the development of young people’s numeracy.


The National Numeracy Review Report (National Numeracy Review Panel, 2008) points out that knowledge of mathematics does not guarantee that learners become numerate. For students to develop numerate behaviour they have to have opportunities to apply mathematical skills in a range of contexts and learning areas. The Report states that:

While the major responsibility for the enhancement of numeracy resides within school mathematics, numeracy outcomes for students will be enhanced by an across the curriculum focus (2008, p.6).

**United Kingdom**

In the UK the National Numeracy Strategy (NNS) was launched in 1998 and has been formally implemented in classrooms since September 1999. The strategy was designed to provide students with a solid foundation in Mathematics and sets
ambitious targets for raising standards in these key skills. It also prescribed a daily
three part mathematics lesson for implementation in all schools. Interestingly, the
most recent iteration of the Primary Framework (DfES, 2006), on which the National
Numeracy Strategy is based, uses the term mathematics rather than numeracy to
describe the recommended learning content for improving students' numeracy
knowledge and skills.

As Hastings (2006, p.1) points out: the Government seems to use the terms
"mathematics" and "numeracy" almost interchangeably. He goes on to state that:

Numeracy is usually defined as the ability to use maths at the level needed to
get by in everyday life. But opinions differ as to what this means. To some
people it means only a command of basic addition, subtraction, division and
multiplication. Others argue that the mathematical skills needed in everyday
life stretch well beyond simple arithmetic. Personal finances, for example, are
increasingly complex: making an informed choice between a variable, capped
or fixed-rate mortgage requires more than just a rudimentary grasp of
percentages. There's also the question of how we make sense of the ever-
growing mass of statistical information available on the internet and in the
media. Want to make up your own mind on global warming? Then it helps to
know about statistical margins of errors and the difference between mean and

Notwithstanding this lack of clarity about the differences between numeracy and
mathematics, it appears the ability to learn mathematical content and to use it in
other learning areas and real life contexts (numeracy) is viewed as important in the
UK. The description of the mathematics curriculum in the renewed Primary
Framework for Literacy and Mathematics (2006) highlights that mathematics learning
should extend beyond mathematics lessons and that students should be provided
with opportunities to:

- practise and use mathematical knowledge and skills in different learning areas
  and contexts
- apply more complex mathematical learning in authentic contexts where they
  use higher-order thinking skills such as reasoning and problem solving
- recognise patterns and relationships in problem solving and reasoning
A report by Ofsted (2008) based on an inspection of mathematics classes in 192 schools in England states that:

_The content of the mathematics curriculum in most of the schools surveyed was age appropriate. However, the majority of students had too few opportunities to use and apply mathematics, to make connections across different areas of the subject, to extend their reasoning or to use information and communication technology (ICT). Higher-attaining students were not always challenged enough in lessons. Links with other subjects were insufficient_ (Ofsted, 2008, p.6).

More recently, the Department for Children Schools and Families (DfCSF) have developed a set of functional skills standards in English, Mathematics and ICT. They are piloting the teaching of these standards in schools across the UK. The notion of students in the secondary years developing a set of functional skills has its origins in the Tomlinson report (DfES, 2004). Tomlinson argued that it was possible for young people to achieve grade C and above in GCSE English and Mathematics without having a satisfactory standard of literacy or numeracy (DfCSF, 2007, p.7).

The mathematics standards aim to support students in the secondary years to use their mathematics skills in a range of contexts and for a variety of purposes. The DfCSF states:

_It is important to recognise that all mathematics can be used in these ways, and that teachers cannot know what mathematics their learners will use as they move through their lives. This means that we cannot identify a curriculum core that every learner will use. Instead, and much more powerfully, learners should be taught to use and apply the mathematics that they know and have learned, and to recognise when they need to develop additional skills_ (2007, p.22).

Therefore, rather than prescribing new topics or concepts that should be taught, the new programs of study specify the types of learning opportunities that students will need if they are to be numerate. An extract from the standards best exemplifies the approach. At level 1 the curriculum should provide students with the opportunities to:
• understand practical problems in familiar and unfamiliar contexts and situations, some of which are non-routine
• identify and obtain necessary information to tackle the problem
• select and apply mathematics in an organised way to find solutions to practical problems for different purposes
• use appropriate checking procedures at each stage
• interpret and communicate solutions to practical problems drawing simple conclusions and giving explanations (DfCSF, p.20).

United States

In the US the term quantitative literacy is frequently used instead of numeracy, though both terms are used to denote the individual's capacity and disposition to apply mathematical knowledge and skills in everyday activities (Steen, 2001; DEECD, 2009, p.7). In recent years there has been increasing interest in the relationship between numeracy and mathematics and Steen (2001) makes a clear statement about this:

Numeracy and mathematics should be complementary aspects of the school curriculum. Both are necessary for life and work, and each strengthens the other. But they are not the same (2001, p.15).

Steen argues that quantitative literacy is a set of knowledge, skills and dispositions about using and applying mathematical content across all areas of the school curriculum and in many contexts in the wider community. Steen also recognises the complex array of knowledge, skills and dispositions that are required if students are to be numerate in a world inundated with numbers, and which makes ever increasing demands on the individual's capacity for quantitative reasoning. She maintains that the following skills are required for effective quantitative literacy.

• Arithmetic: Having facility with simple mental arithmetic; estimating arithmetic calculations; reasoning with proportions; counting by indirection (combinatorics).
• Data: Using information conveyed as data, graphs, and charts; drawing inferences from data; recognizing disaggregation as a factor in interpreting data.
• Computers: Using spreadsheets, recording data, performing calculations, creating graphic displays, extrapolating, fitting lines or curves to data.

• Modeling: Formulating problems, seeking patterns, and drawing conclusions; recognizing interactions in complex systems; understanding linear, exponential, multivariate, and simulation models; understanding the impact of different rates of growth.

• Statistics: Understanding the importance of variability; recognizing the differences between correlation and causation, between randomized experiments and observational studies, between finding no effect and finding no statistically significant effect (especially with small samples), and between statistical significance and practical importance (especially with large samples).

• Chance: Recognizing that seemingly improbable coincidences are not uncommon; evaluating risks from available evidence; understanding the value of random samples.

• Reasoning: Using logical thinking; recognizing levels of rigor in methods of inference; checking hypotheses; exercising caution in making generalizations (Steen, 2001, pp.16–17).

Steen emphasises that quantitative literacy is inseparable from context and, that numeracy has no special content of its own, but inherits its content from its context (2001, p.17).

Australia

In 2006, a review was commissioned by the Council of Australian Governments (COAG) into the teaching, learning and assessment practices that lead to improved numeracy outcomes for students. As well as undertaking extensive consultation with state education systems and sectors, the National Numeracy Review Report Panel conducted an extensive review of current national and international research and received submissions from a range of targeted stakeholders.

One of the key findings of the National Numeracy Review Report (2008) was that there needs to be greater clarity around the definitions of numeracy and mathematics. The Panel states:
for many members of the broader community and indeed for many teachers and policy makers, the term ‘numeracy’ is used more or less synonymously with mathematics, or even with the ‘basics’ of mathematics, particularly in the context of public commentary about ‘numeracy standards’ (2008, p.5).

However, as the Panel point out, this raises the questions about which basics and what standards? Further, there is evidence that the numeracy knowledge and skills that people use in work and real life situations are often very different to the written proficiencies characteristic of much school mathematics. Importantly, the Panel acknowledge while schools are unable to always offer real world settings, they can provide access to a range of different contexts where numeracy knowledge and skills can be used in all learning areas. The National Numeracy Review Report Panel maintain while the major responsibility for the enhancement of numeracy resides within school mathematics, numeracy outcomes for students will be enhanced by an across the curriculum focus (2008, p.6).

It was recommended by the Panel that numeracy programs should include:

- mathematical content, that is, the concepts, procedures and skills which comprise what we think of as ‘school’ mathematics
- a repertoire of strategic mathematical processes, appreciations and dispositions needed to choose and use mathematics to solve familiar and unfamiliar problems
- opportunities to develop and apply mathematical knowledge in a range of situations, both familiar and unfamiliar, in order to develop an understanding of the way in which contextual features can determine the appropriateness and usefulness of particular mathematical approaches (2008, p.9).

The National Numeracy Review Panel note that given the demands of mathematics in work situations, students will need a deep understanding of the real number system; statistical, geometrical and algebraic thinking as well as experience in using charts, data tables and spreadsheets. However, they also point out that students must have experience grounded in practical situations of making contextualized judgements about levels of accuracy. They need the knowledge, skills and confidence to determine when to approximate, to problem solve and communicate
findings, to evaluate the reasonableness of answers and to adapt their learning and learn new skills. They also need opportunities to work in teams (2008, p.10).

In summarising their ideas on what should be in a numeracy curriculum the Panel state that:

> To be truly numerate involves considerably more than the acquisition of mathematical routines and algorithms, no matter how well they are learned. Students need to learn mathematics in ways that enable them to recognise when mathematics might help to interpret information or solve practical problems, apply their knowledge appropriately in contexts where they will have to use mathematical reasoning processes, choose mathematics that makes sense in the circumstances, make assumptions, resolve ambiguity and judge what is reasonable in the context (2008, p.11).

Additionally, the National Numeracy Review Report Panel (2008, p.32) recognise that effective numeracy programs include learning content on the literacy demands of numeracy. This includes content on explanations of terms frequently used in mathematics such as factorise and evaluate. The Panel cite work conducted by DiGisi & Fleming (2005, p.49) who contend that students need to read and understand three types of vocabulary: math vocabulary (square yard, circumference, symmetry, integers), procedural vocabulary (factorise, estimate) and descriptive vocabulary which provides the context for the math problems. Numeracy programs also need content on the generic structures (including structure, sentence conventions and language) for communicating orally and in writing mathematical understandings (National Numeracy Review Report Panel, 2008, p.33).

To meet the diverse needs of students, in particular multilingual learners, numeracy programs should include learning activities that allow for exploration and discussions of the socio-cultural implications of mathematical language. In their evaluation of a Mathematics project in Indigenous contexts Erebus International point out:

> Mathematics education is embedded in a particular cultural context. Mathematics is a socially constructed way of encoding, interpreting and organising the patterns and relationships emerging from the human experience of physical, spiritual and social phenomena, and learning mathematics is therefore a form of enculturation. The mainstream school
Mathematics curriculum is based on what has been described as the Western Mathematics paradigm. There are many differences between this and the framework in which Aboriginal mathematics is embedded (Erebus, 2007, p.19).

If literacy programs are to be effective for Indigenous students, teachers need to provide extensive oral activities and sufficient time for students to explore mathematical concepts and skills and their associated intercultural understandings in both Western and Indigenous culture, allowing for discussion of concepts in both the first and second languages (Erebus, 2007; National Numeracy Review Report Panel, 2008).

Summary

Given that numeracy is about students’ capacities, confidence and dispositions to apply mathematical knowledge and skills to their everyday lives, it is difficult to be specific about the learning content for effective numeracy programs. Arguably, each individual student could require different knowledge and skills depending on their work and life situations. However, the research does indicate that there are some common elements or critical features that should be evident in effective numeracy programs if students are to have the capacity, confidence and disposition to apply and/or adapt mathematical understandings and skills learned at school.

These include:

- mathematical content, that is, the concepts, procedures and skills which comprise the learning area of mathematics
- a repertoire of mathematical proficiencies to choose and use mathematics to solve familiar and unfamiliar problems
- a systematic instructional sequence of learning experiences
- opportunities for mathematical concepts, procedures, skills and proficiencies to be applied across a range of learning areas and in the wider community
- opportunities to make links between concepts within and across the strands of mathematical content
- a range of examples and practice tasks that extend beyond repetitive routines
- additional content on the linguistic understandings that are required for numeracy
• content on the intercultural understandings that are implicit in the knowledge, understandings and skills required for numeracy.

**Teaching and learning strategies**

In recent years, there have been a number of significant large-scale studies or meta-analyses of research both in Australia and overseas about:

• the characteristics of effective mathematics/numeracy teachers
• the teaching and learning strategies that lead to enhanced mathematics/numeracy outcomes for students.

While this review has already established that numeracy is not the same as mathematics, they are closely related. Being numerate relies on students having the capacity and disposition to use mathematical knowledge and skills in a range of everyday contexts. Therefore, it is reasonable to expect that findings from studies about effective teaching and learning practices in mathematics are relevant to the teaching and learning practices required for students to demonstrate numerate behaviour. However, Siemon, Virgona and Corneille (2001) observe: *while ‘good’ mathematics teaching is necessary to numeracy improvement, it is not sufficient*. This is because numeracy learning is context specific while mathematical learning deals more with abstract reasoning and generalities.¹

As Manaster points out:

*This divergence of emphasis [between mathematics and numeracy] does not require completely separate instructional paths, but it does require that teachers give adequate and separate attention to the goals of each. At times, numbers and abstractions of number systems should be studied to understand and master efficient computational algorithms and to see what properties they satisfy. At other times, elements of this abstract knowledge need to be applied to understand models of complex concrete situations* (in Steen, ed., 2001, p.70).

¹ Manaster defines mathematics as the science of numbers and their operations, interrelations, combinations, generalizations, and abstractions and space configurations and their structure, measurement, transformations, and generalizations (2001, p.68).
There will be differences in where these teaching and learning practices will be used—to be numerate requires students to be able to have more than just knowledge and skills in mathematics but also to apply them in a range of real world contexts. As a consequence, numeracy should be taught in all learning areas and all teachers should have some degree of proficiency in these teaching and learning practices.

**United Kingdom**

In 1997, Askew et al. examined the link between teachers' practices, beliefs and knowledge and student learning outcomes in the development of numeracy knowledge and skills with students aged five to eleven. From an initial sample size of 587 schools, they selected a sample of 90 teachers based on rigorous evidence of their capacity to improve student achievement (1997). Of these, they chose a smaller sample for close evaluation and monitored beliefs and practices through interviews and classroom observation.

The study found that there were generally three categories of teachers operating in numeracy classrooms. The most effective of these was the group they classified as connectionist teachers, evidenced by the fact that their students made the greatest gains in learning outcomes. Connectionist teachers viewed mathematics as going beyond the recall of number facts, developing in their students an awareness of connections and relationships in mathematical understandings. Their students developed flexible mental strategies for handling number calculations. These strategies gave students the knowledge, skills and confidence to tackle calculations for which methods had not been taught. The connectionist orientated teachers placed strong emphasis on challenging all students irrespective of ability (Askew et al., 1997).

Askew et al. (1997) state that connectionist teacher lessons were often characterised by:

- a high degree of focused discussion between the teacher and various groupings of students—whole class, small group and individual students
- discussions which encouraged students to share methods of carrying out calculations and problem solving
• opportunities for students to explain their thinking processes rather than simply providing answers
• opportunities for students to apply mathematical skills
• students who were confident in sharing their thinking and challenging the assumption of others
• the challenge of a problem providing the context for students to learn new skills (Askew et al., 1997).

In 2008, Ofsted (The Office for Standards in Education) in the UK, published a comprehensive report that synthesised findings about effective mathematics teaching based on evidence from school inspection visits in 192 schools across England between April 2005 and December 2007. Key findings from this report mirror many of the themes that emerged from the Askew et al. study. Ofsted asserts that:

The best teaching was rooted in developing students’ understanding of key concepts. It was inclusive in terms of ensuring that all students made substantial progress, no matter what their starting points. In the outstanding lessons, the teachers had high expectations of students’ enjoyment and achievement. They made conscious efforts to foster a spirit of enquiry, developing students’ reasoning skills through approaches that saw problem-solving and investigation as integral to learning mathematics. They checked that everyone was challenged to think hard and they adapted how they were teaching to achieve this (2008, p.12).

The report goes on to describe a comprehensive set of features for effective mathematics teaching. In regard to lesson structure and general teaching approaches the report points out that:

• lesson objectives involve understanding and make what is to be learned in the lesson very clear
• students are clear about what they are expected to learn in the lesson and how to show evidence of this
the lesson forms a clear part of a developmental sequence and students recognise links with earlier work, different parts of mathematics or contexts for its use

- teaching successfully focuses on each student’s learning
- teachers introduce new terms and symbols meaningfully and expect and encourage correct use (meta language) (Ofsted, 2008, p. 13).

Ofsted (2008) identifies the following instructional strategies and features of assessment for learning as important in effective mathematics teaching.

- Teachers (and students) have a good grasp of what has been learnt judged against criteria that they understand; this is shown through student discussion, reflection, oral or written summaries, and ascertained by the teacher’s monitoring throughout the lesson.
- When offering answers or accounts, the teacher expects students to give explanations of their reasoning as well as their methods.
- The teacher listens carefully and interprets students’ comments correctly, building on students’ contributions, questions and misconceptions to aid learning and extend thinking, flexibly adapting to meet needs and confidently departing from plans.
- Students develop independence and confidence by recognising when their solutions are correct and persevering to overcome difficulties because they expect to be able to solve problems.
- Teaching assistants know the students well, are well briefed on the concepts and expected misconceptions, and provide support throughout the lesson that enhances thinking and independence.
- Teachers’ marking identifies errors and underlying misconceptions and helps students to overcome difficulties (Ofsted, 2008, pp. 13–14 and pp. 45–49).

The nature of the learning task is also seen as vital in effective mathematics teaching. The report states the teacher provides work that:

- challenges all students as it is informed by teachers’ knowledge of students’ learning
- requires thinking and reasoning and enables students to fully understand objectives
includes non-routine problems, open-ended tasks and investigations to develop the broader mathematical skills of problem solving, reasoning and generalising

provides opportunities to extend understanding, such as through links to other subjects, more complex situations or previously learned mathematics

includes practical work, discussion and makes effective use of ICT (Ofsted, 2008, pp.27–39).

Ofsted also notes that the best teachers are adept at developing a positive learning environment. They point to the following as evidence.

- Teachers ensure all students participate actively in whole-class activity and partner or group discussions.
- Respect is conveyed for students’ contributions so that many offer right and wrong comments.
- Students naturally listen to and respond to each other’s comments showing engagement with them (Ofsted, 2008, pp.13–14).

Assessment

Ofsted notes in its report that assessment for learning practices continues to be a relatively weak area in mathematics teaching. The researchers believe its potential for checking on and promoting students’ understanding was not employed by teachers often because of shortcomings in their subject knowledge or pedagogic skills (2008, p.6).

Assessment for learning has been a key focus of instructional practice in the UK and other countries since Black and Wiliam’s landmark review of a wide range of studies on the use of assessment. They found that strengthening the practice of formative assessment produces significant and often substantial learning gains (1998, p.3).

Black and Wiliam highlight a number of formative assessment practices that when used in combination have the potential to maximise learning.

These include:

- establishing and sharing with students key learning goals with clear criteria for performance
- regular monitoring of their progress using effective questioning techniques
• self and peer assessment
• providing specific feedback on strengths and weaknesses
• realignment of learning to address gaps in knowledge and skills (Black and Wiliam, 1998, pp.7–16).

Since the publication of these initial findings, a substantial body of evidence has emerged about the most effective ways to use assessment for learning strategies in mathematics teaching and learning. In relation to questioning techniques, Wiliam points out that:

*Teachers tend to use questions as a way of directing the attention of the class, and keeping students ‘on task’, by scattering questions all around the classroom. This probably does keep the majority of students ‘on their toes’ but makes only a limited contribution to supporting learning* (2005, p.23).

Wiliam cites a substantial body of evidence which demonstrates that the use of more effective questioning techniques has produced measurable increases in learning. He states that a more valuable strategy is for teachers to conduct an extended exchange with a single student involving a series of questions that probe for deeper understanding. This prompts a more sophisticated level of discussion about the concept. He also recommends increasing the time between the end of the student’s answer and the teacher’s evaluation from the average ‘wait time’ of less than a second to three seconds. Techniques such as asking students to generate their own questions, and teachers making statements and providing opportunities for open discussion around the statement, often allow teachers to identify misconceptions and to probe for deeper learning (Wiliam, 2005, p.24).

Wiliam emphasises the importance of timely, specific feedback on the task to enhance learning. He states: *Feedback to learners should focus on what they need to do to improve, rather than on how well they have done, and should avoid comparison with others* (2005, p.28). The importance of feedback in effective learning is confirmed by Hattie. He maintains that feedback which is timely, specific, clear and directed to the level or nature of task performance is very powerful in enhancing learning, especially if it is coupled with effective instruction (Hattie, 2009, p.178).
Wiliam (2005, pp.28–30) also supports the practice of providing students with the performance criteria required in assessment tasks and giving them adequate time to explore what successful achievement might look like. Providing samples of student work where students can discuss the performance criteria, and the extent to which they have been met, improves students’ learning outcomes.

Moreover, Wiliam contends that involving students in assessing their own learning and/or working with teachers and/or peers to reflect on their learning provides opportunities for students to gain deeper understandings of:

- where the learners are in their learning
- where they are going
- how to get there (2005, p31).

**United States**

In recent years there have been a number of significant research studies on how to improve mathematics teaching in the US, largely prompted by a recognition that: *international and domestic comparisons show that American students have not been succeeding in the mathematical part of their education at anything like a level expected of an international leader* (National Mathematics Advisory Panel, 2008).

In 2006, a Panel of experts was established in response to an executive order from the President to examine all the best scientific evidence on all aspects of mathematics teaching and to provide advice on improvements needed in mathematics education. In accordance with these directions, the Panel only considered research that was rigorous and whose findings could be generalised to the extent that it could shape educational policy.

In their findings, the National Mathematics Advisory Panel state that research does not support the exclusive use of either ‘student-centred’ or ‘teacher-directed’ instruction (2008, p.45). There was evidence that both could lead to improved outcomes. However, the Panel acknowledge that explicit instruction is important for students who have mathematical difficulties, in particular, to improve their performance with word problems and computation. This explicit instruction entails

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2 Aspects of mathematics teaching that were examined by the panel included Conceptual Knowledge and Skills, Learning Processes, Instructional Practices, Teachers and Teacher Education, and Assessment (National Mathematics Advisory Panel, 2008, p.xvi).
teachers providing clear models for solving problems. For example, providing students with opportunities for extensive practice in the use of newly learned strategies and skills, providing opportunities to think aloud, and providing extensive feedback. However, the Panel point out this does not mean that all of a student’s mathematics instruction be delivered in an explicit fashion (2008, p.48).

The National Mathematics Advisory Panel note that the following features of effective instructional practice in mathematics have demonstrated positive effects on student outcomes. These include:

- sufficient and appropriate practice to develop automaticity
- a variety of cooperative learning approaches, including Team Assisted Individualisation (TAI), which have been shown to improve students’ computation skills
- regular use of formative assessment which improves their students’ learning, especially if teachers have additional guidance on using assessment to design and individualise instruction
- the use of ‘real world’ contexts to introduce mathematical concepts

In recent years, Slavin and Lake (2008) and Slavin, Lake and Groff (2010) have conducted two important studies on effective programs in mathematics. The first was a best evidence synthesis of effective programs for students in the elementary years and the second focused on mathematics programs for students in the middle and high school years. Both studies employed a similar methodological approach where all types of programs intended to enhance mathematics achievement were placed on a common scale to identify teaching and learning strategies likely to make a difference in achievement.

The most striking conclusion from these studies was that it was not the content of the mathematics program but rather the way in which it was taught that had a significant influence on improving student outcomes. Both studies found that cooperative learning where students work in pairs or small teams and are rewarded based on the learning of all team members, was effective (Slavin & Lake, 2008; Slavin, Lake & Groff, 2010). For students in the elementary years Team Accelerated Instruction—this combines cooperative learning and individualization—demonstrated moderate
evidence of effectiveness. Additionally, programs that indicated some evidence of effectiveness include those that focus on improving teachers’ skills to introduce mathematics concepts effectively, classroom management, motivation, and effective use of time (Slavin & Lake, 2008, pp.481–482). In the study that explored effective programs for students in the middle years, Slavin et al. point out that strategies found to be effective with minority social or ethnic groups, including second language learners, tend to be effective with all groups (2010, p.887).

**New Zealand**

One of the most significant pieces of research to emerge in recent years on mathematics teaching and learning practices is the report, *Effective Pedagogy in Mathematics/Pāngarau: Best Evidence Synthesis Iteration [BES report]*. This report is one of a series of best evidence iterations developed in New Zealand that uses rigorous scientific evidence to guide the ongoing improvement of education at the national level. The report was designed to identify from the research the pedagogies that would enhance student mathematical proficiencies, in particular for learners form diverse backgrounds. Large-scale studies and meta-analyses and small site based case studies that cover the diversity of learners and are relevant for New Zealand were examined in the research. The findings are comprehensive and cover all aspects of mathematics teaching across all school age levels.

The report’s authors, Anthony and Walshaw, make extensive recommendations about pedagogy and the types of learning tasks that evidence has shown will improve student learning outcomes in mathematics. According to the authors, planning appropriate learning sequences is an essential role for teachers. Connecting tasks to learners’ existing proficiencies and knowledge and using them to scaffold new learning are also vital components of effective pedagogy (Anthony and Walshaw, 2007a, pp.109–111).

The importance of explicit instruction about mathematical language is emphasised in Anthony and Walshaw's report. They point to a range of research that has shown that all students, in particular those from diverse language or cultural backgrounds, need to be *explicitly taught the conventions and meanings associated with mathematical discourse, representation, and forms of argument* (2007a, p.69).
Anthony and Walshaw also examined research associated with teaching multilingual learners. They cite studies with Samoan and Tongan learners by Fasi (1999) and Latu (2005) who found that some frequently used mathematical terms have no corresponding concepts in a student’s home language. They argue that extensive scaffolding needs to occur to connect the underlying meaning of a concept in English with the students’ home language. This often involves code switching between first or home languages and English.

Additionally, Anthony and Walshaw highlight an extensive body of research that has demonstrated the benefits of asking students to justify and/or explain their thinking. The researchers state that a key characteristic of effective pedagogy is when teachers ask students to articulate their thinking, elaborating on their strategies and/or reasoning (2007a, pp.72–74). They believe that ‘pressing’ for understanding is an important aspect of quality mathematics pedagogical practice. Similarly, they promote the development of the skills of mathematical argumentation where students take and defend positions against alternative views (Anthony and Walshaw, 2007a, pp.120–124). Moreover, Anthony and Walshaw concur with Black and Wiliam (1998) about the importance of using assessment for learning strategies to guide instruction (2007b, pp.17–18).

Anthony and Walshaw stress that teachers need to attend to the different cultures and social backgrounds of students to support all students in developing mathematical identities that are confident and positive. Providing expectations about how students can contribute and respond to each other in classroom discussions offers the supportive and productive learning environment that students need for mathematical thinking, reasoning and problem solving. They also point out that students need opportunities to work both independently and collaboratively (2007a, pp.58–67).

Anthony and Walshaw also include an extensive discussion on the types of mathematical tasks required for effective learning. According to them, students should have opportunities to practise a wide range of examples and practise tasks that exemplify structural similarities and differences so they can construct generalisations. There should also be opportunities for students to explore authentic applications in all learning areas. However, it is important that the contexts are
familiar or explained to all students and that the context does not obscure the mathematical purpose of the problem (2007a, p.114).

A key finding in their report that is of particular relevance to teachers in the NT is that:

While not all mathematical tasks need originate from students’ cultural experience, it is necessary that embedded contexts are accessible to all students. Embracing culturally contextualised pedagogy is not, however, simply a matter of incorporating ethnic symbols and artefacts into tasks.

Anthony and Walshaw mention the work of Sullivan, Mousley and Zevenbergen’s (2003) research project who found that a more sound pedagogical approach is where the teacher makes explicit those hidden aspects of pedagogy that can inhibit student participation in open-ended contextually-based tasks (2007a, p.115).

Anthony and Walshaw also point out that mathematical tasks should:

- be challenging for students but they should also be realistic, offering opportunities for success
- extend students’ understandings and their ability to apply their mathematical strategies in new and different ways and contexts stimulating higher order cognitive activity
- provide the ideal opportunity for ‘mathematical play’ which allows for a range of ‘correct’ responses and a range of ways of achieving those responses
- require students to make multiple connections within and across topics to help them appreciate the interconnectedness of different mathematical ideas and the relationships that exist between mathematics and real life (2007a, pp.82–106).

They support the use of representational artefacts in mathematics learning to bridge the gap between concrete situations and abstract mathematics and favour the use of digital technologies that are embedded in pedagogy rather than an adjunct to it (Anthony and Walshaw, 2007a, pp.132–136).

**Australia**

Over the past ten years a number of important Australian research studies have focused on the teaching and learning of numeracy. One of the first of these, *The
Early Numeracy Research Project, was conducted in seventy Victorian schools from 1999–2001. As part of their project the researchers closely observed a group of teachers that had been identified as highly effective based on evidence from a range of student assessments. Clark et al. (2001, p.13) identified the following characteristics of effective early numeracy teachers.

| Mathematical focus                           | • focus on important mathematical ideas  
|                                            | • make the mathematical focus clear to the children |
| Features of tasks                           | • structure purposeful tasks that enable different possibilities, strategies and products to emerge  
|                                            | • choose tasks that engage children and maintain involvement |
| Materials, tools and representations        | • use a range of materials/representations or contexts for the same concept |
| Adoptions/ connections/ links               | • use teachable moments as they occur  
|                                            | • make connections to mathematical ideas from previous lessons or experiences |
| Organisational style(s), teaching approaches| • engage and focus children’s mathematical thinking through an introductory, whole group activity  
|                                            | • choose from a variety of individual and group structures and teacher roles within the major part of the lesson |
| Learning community and classroom interaction| • use a range of question types to probe and challenge children’s thinking and |
reasoning

- hold back from telling children everything
- encourage children to explain their mathematical thinking/ideas
- encourage children to listen and evaluate others’ mathematical thinking/ideas, and help with methods and understanding
- listen attentively to individual children
- build on children’s mathematical ideas and strategies

| Expectations | • have high but realistic mathematical expectations of all children
|              | • promote and value effort, persistence and concentration |
| Reflection   | • draw out key mathematical ideas during and/or towards the end of the lesson |
| Assessment methods | • collect data by observation and/or listening to children, taking notes as appropriate
|              | • use a variety of assessment methods
|              | • modify planning as a result of assessment |
| Personal attributes of the teacher | • believe that mathematics learning can and should be enjoyable
|              | • are confident in their own knowledge of mathematics at the level they are |
In 2001, Doig undertook a best-practice synthesis of international research, including many Australian projects. He summarised his findings on effective teaching and learning for mathematics by listing a series of teacher responsibilities. He states that teachers should:

- *ensure cultural inclusivity*
- *allow for local differences, for example dialects*
- *make students feel secure, safe and challenged*
- *have an expectation that students can achieve*
- *have a well-structured program, with achievable goals*
- *use a range of learning media*
- *use assessment tasks that are directly related to classroom tasks*
- *identify numeracy problems early*
- *use frequent diagnostic assessment*
- *have a solid period of focused instruction*
- *have lessons with a conceptual focus*
- *make greater use of open-ended questions*
- *give students more time to explore concepts*
- *give students more opportunity to share their strategies*
- *have more emphasis on links and connections between mathematical ideas and classroom mathematics* (Doig, 2001, p.32).

Another important study was commissioned by the Victorian Department of Education, Employment and Training (DEET), the Catholic Education Commission of Victoria (CECV) and the Association of Independent Schools of Victoria (AISV) to provide advice on a co-ordinated and strategic plan for improving the teaching and learning of numeracy in years 5 to 9. Conducted by Siemon, Virgona and Corneille (2001), the research project had a number of stages. Firstly, the researchers implemented ‘rich assessment tasks’ and an extended classroom activity to assess both the students’ knowledge of key, underpinning mathematical ideas and their capacity to apply and communicate this knowledge in context in a sample of 47 schools across Victoria. Secondly, data collected from these assessments and
information about each school’s numeracy policies and programs was used to develop a set of design elements for effective numeracy programs. Finally, to determine what works in numeracy teaching for students in the middle years, these design elements were trialled in a smaller sample of 20 schools.

The research study found that the following teaching and learning activities enhanced numeracy performance in the sample schools.

- Regular and systematic use is made of open-ended questions, games, authentic problems and extended investigations to enhance students’ mathematics learning and capacity to apply what they know.
- Teaching strategies are focused on connections and strategies for making connections.
- Students are actively engaged in conversations and texts that encourage them to reflect on their learning and explain and justify their thinking.
- Special attention is given to the literacy aspects of mathematical texts and representations.
- Learning activities are designed or chosen appropriate to learners’ needs and interests and use a balance of teacher-directed and student-centred approaches.
- Opportunities are provided for meaningful and enjoyable practice of essential knowledge and skills (2001, pp.104–104).

Notwithstanding the important findings from these reports, perhaps the most influential work on numeracy teaching and learning in recent years is the National Numeracy Review Report (2008). The National Numeracy Review Report Panel reviewed relevant submissions and recent national and international research on mathematics and numeracy education to determine the most effective, evidence-based teaching and learning practices.

The Panel conclude:

While there are particular characteristics associated with more effective teachers, formulating an adequate and quantifiable definition of quality teaching is challenging – what works in one context may not work in another. Measuring the extent to which teachers have caused student gains on
assessments is also difficult. This suggests that care should be taken in developing measures of quality teaching.

Nevertheless, they concede there is broad support for approaches that employ cooperative, collaborative, dialogic strategies in teacher-led, peer and individually driven learning.

The Panel recommend that:

Greater emphasis be given to providing students with frequent exposure to higher-level mathematical problems rather than routine procedural tasks, in contexts of relevance which provide increased opportunities for students to explain their thinking and discuss alternative solutions in numeracy programs.

The National Numeracy Review Report Panel acknowledge that language can provide a barrier to effective numeracy learning. They include an extensive discussion of the importance of literacy in mathematics in the report. The Panel believe mathematical literacy refers to students being able to access the mathematics in words and to make sense of the context and clarify what is required (2008, p.33). It also refers to students being able to communicate in mathematics, for example, to explain their solutions and write investigations. The Panel propose that the language and literacies of mathematics be explicitly taught by all teachers of mathematics (2008, pp.32–34).

The Panel confirm that formative assessments are a valuable tool for monitoring students' learning, providing feedback and guiding future instruction. They point out that data from large-scale tests can be used to improve teaching and enhance student learning outcomes (2008, p.34). The panel also recognise the importance of teachers using diagnostic tools, including interviews, to identify levels of student achievement and to inform instructional design (2008, p.42).

The panel referred to a wide body of research on the use of digital technologies in schools. They acknowledge evidence from the literature that highlights the many benefits of using such technologies to support or deepen learning or to assist in learning delivery. Nonetheless, they note that the effectiveness of digital technologies in mathematics classrooms often depends upon the teachers’ beliefs about its use as well as their ability to embed its use in their practice. They point out
that digital technology is most effective when it supplements or enriches quality pedagogical practice (2008, pp.44–46).

**Indigenous Students**

The National Numeracy Review Panel drew particular attention to strategies required for improving numeracy outcomes for Indigenous students. They believe when teaching mathematics to Indigenous students who have English as an additional language or dialect there is a critical need to recognise *the importance of spoken language as the foundation of all learning* (2008, p.58).

Like Anthony and Walshaw (2007a), the panel recognise that students must be provided with opportunities to connect mathematical concepts with existing cultural understandings by exploring them in the first/home language and in English. They state that:

> Teachers should not assume that Indigenous students will share their understanding of the English words and concepts that they use. In some traditional/non-urban or rural Indigenous contexts for example, numbers may be familiar to students in a nominal sense through everyday contexts such as numbers on vehicle number plates or football jumpers, but not in a cardinal sense, such as for comparing quantities (eg I have 20 pens and he has 25). Students may need to be explicitly taught the use of numbers for comparison to provide a context for learning how to count to determine ‘how many’ (2008, p.58).

The National Numeracy Review Report Panel cited two programs for Indigenous students that have produced improved learning outcomes (Erebus International, 2007; Meaney, 2001) and note the following teaching and learning practices were apparent in the programs. These included teachers:

- valuing the culture, language and the richness of what Indigenous students bring with them to the classroom
- having high expectations of students and their learning
- valuing different approaches to learning and recognising and valuing different learning pathways
- recognising the importance of building strong relationships with their students
• using community and Indigenous partnerships to create a culturally and contextually aligned learning programme
• acknowledging the critical importance of having first language speakers in the classroom to assist learners to elaborate and scaffold their mathematical thinking
• using relevant and meaningful contexts to ‘situate’ the learning in students’ lives
• adopting strategies to deal with hearing loss, homework incomplection, and absenteeism
• paying particular attention to socio-cultural differences in learning styles in the delivery of the mathematics curriculum, for example, through teaching students how to use talk for mathematical learning, to take risks in their learning and by teachers adapting their questioning style (2008, pp.55–56).

The Supporting Indigenous Students’ Achievement in Numeracy (SISAN) Project 2003–2004 was designed to explore the impact of authentic (rich) assessment tasks on the numeracy outcomes of middle years Indigenous students in a targeted group of remote schools. Overall the project found that the use of authentic (rich) tasks has the potential to support Indigenous students in middle years in remote settings. However, the researcher acknowledged this is a significant task that requires the involvement of Indigenous and non-Indigenous teachers, schools and community leaders over an extended period of time. Teaching strategies that were found to be effective include:

• using concrete materials, teaching aids
• reading, re-reading or restating the question using the first language
• modelling of what the task involves
• discussion of the situation and elaborating on the context
• working with a smaller group
• the use of diagrams or models to explain tasks
• choosing appropriate/meaningful contexts to facilitate student engagement
• scaffolding the language and/or the task to support comprehension
• the provision of carefully targeted scaffolding that alerted students to key features of the task
• the use of focused questions to redirect students to the task
• careful changes to the language demands of the task to make it more accessible
• modifications to the social setting of the task to more closely match the students’ experience
• careful attention to response modes and layout (DEST, 2005, pp.34–37).
Students with learning difficulties

The National Numeracy Review Panel also explores effective strategies for teaching mathematics to students with learning difficulties. They conclude that while withdrawal programs can be effective, it is preferable for support teachers to work with classroom teachers using differentiated pedagogies. They contend this strategy often allows for more efficient use of resources and provides opportunities for teachers to support each other’s professional learning (National Numeracy Review Report Panel, p.63). This approach is supported by the findings of Meiers et al. in their evaluation of the Getting it Right - Literacy and Numeracy Strategy (GiR-LNS). The focus of GiR-LNS was on improving literacy and numeracy teaching in the early years of schooling, especially for students at risk of not making satisfactory progress. Meiers et al. found that:

Specialist teachers working shoulder to shoulder with teachers in their classrooms had a positive impact on the capacity of teachers to select, apply and develop diagnostic, formative and summative student assessment strategies and instruments so that they were better able to focus on individual learning needs in literacy and numeracy (2005, p.2).

The use of fine grained data to identify and diagnose students’ learning needs and to plan explicit teaching approaches to address the diversity of students’ needs was also a feature of effective teacher pedagogy for students who were at risk of not reaching numeracy benchmarks (Meiers et al. 2005, p.2).

Louden et.al (2000, p.11) scanned national and international research and worked with teachers in a sample of case study schools across Australia to determine appropriate pedagogical practices for students struggling with learning numeracy knowledge and skills.

They conclude that teachers should:

- scaffold learning, making sure instructions are clear and understood
- provide hands-on activities
- move from the concrete to the abstract
- encourage students to verbalise the process used in calculating or solving a problem
• focus on the big ideas to make learning other concepts more meaningful and easier
• teach strategies for solving problems
• introduce new content gradually
• teach different strands together to support students in making connections and generalisations
• provide regular feedback
• provide opportunities for students to practise skills and strategies and to review them (Louden et al., 2000, pp.11–12).

The QuickSmart® Numeracy program is a theory-based intervention approach to the assessment, diagnosis and remediation of basic academic skills in numeracy. The program aims to increase automaticity in basic number facts for numeracy. When students develop automaticity they free up their working memory for use on more complex tasks. Further, students develop greater self-confidence and are more likely to take risks in problem-solving and engaging in classroom activities. This increase in automaticity and self-confidence leads to enhanced student achievement in learning outcomes.

An unpublished report on the implementation of the QuickSmart® Program in the NT states:

*Student learning achievement data for the QuickSmart® Numeracy Program between 2005–2009 clearly indicate that it has had a significant effect in ‘narrowing the gap’ in achievement of mathematics outcomes for the QuickSmart® student cohort (middle years students not meeting the Minimum National Numeracy Standards) and the comparison student cohort (comprised of average-ability students of the same age). Student learning data collected from the pilot QuickSmart® Literacy Program 2008–2009 has revealed similar results. There is some evidence that QuickSmart® students have increased performance in system numeracy assessments (NTDET, 2010, p.27).*

**Summary**

Defining a set of characteristics or features of effective instructional strategies for teaching and learning numeracy knowledge and skills is problematic as what works in one context with a particular group of students may not be successful in different
educational contexts. Often, what makes the difference between ‘ordinary’ and effective teaching is the practitioner’s ability to select and integrate strategies for a particular learning purpose or context according to students needs. Moreover, the skills of weaving strategies together to present a challenging and coherent learning experience for students are usually developed after considerable reflection on and analysis of student achievement and associated teaching practices. Nevertheless, there are some reoccurring messages in the research about effective teaching and learning practices that have resulted in improved numeracy outcomes for students. There is general consensus in the literature that effective teaching and learning of numeracy knowledge and skills:

- provides learning experiences (lesson plans and teaching and learning sequences) with coordinated sequences, structures and routines
- presents clear, specific goals for learning with performance criteria
- links to and builds on existing knowledge, skills and understandings
- provides activities for exploring mathematical language by using non linguistic representations of terms and concepts, in particular for multilingual learners
- includes strategies for understanding the language and literacy requirements of numeracy
- models and encourages the use of developmentally appropriate mathematical terms or metalanguage
- includes explicit or direct instruction
- uses constructivist or inquiry approaches which have been preceded by explicit or direct instruction to ensure learners have basic knowledge, skills and understandings
- scaffolds learning by breaking down concepts or problems into component parts to allow for ideas to be understood
- encourages modelling of methods or processes accompanied by detailed explanations
- provides opportunities for learners to practise skills to develop automaticity and to increase fluency, accuracy and efficiency
- includes questioning techniques that elicit feedback on student knowledge and understandings to realign instructional strategies according to responses
- involves a range of higher order learning strategies
includes strategies for providing timely and appropriate feedback
provides opportunities for learners to apply knowledge, skills and understandings in a range of contexts within classes, in other learning areas and in everyday contexts
provides opportunities for learners to engage in problem solving and investigations
includes opportunities for cooperative learning according to the requirements of the learning tasks and the needs of the learner
provides opportunities for differentiated instruction, including English language instruction for multilingual learners
includes strategies for intensive instruction for learners with particular learning needs
embeds the use digital technologies in learning
operates in a socially supportive and productive classroom environment.

Professional learning

Almost every report or research study examined for this literature review emphasised that effective professional learning is a critical factor in improving teacher efficacy and student learning outcomes. To address this issue, the National Numeracy Review Report Panel recommend the provision of structured professional learning programs that provide mathematics teachers with both content knowledge and pedagogical content knowledge to ensure quality numeracy and mathematics programs. This recognition that teachers need both discipline based knowledge and pedagogical content knowledge was reiterated in much of the research (Askew et al. 1997; Louden et al., 2000; Anthony and Walshaw, 2007; Ofsted, 2008; National Mathematics Advisory Panel, 2008). There is also recognition that professional learning programs should be extended to include all teachers so they can teach the numeracy knowledge and skills required in their particular learning areas (Steen, 2001; National Numeracy Review Report Panel, 2008).

In addition, Askew et al. (2007) state that highly effective teachers of numeracy knowledge and skills were much more likely than other teachers to have undertaken ongoing mathematics specific professional learning over an extended period, and generally perceived this to be a significant factor in their development. They note that
mentoring, coaching and opportunities to work in professional learning communities were key features of professional learning undertaken by highly effective teachers. Likewise, Pegg, Lynch and Pannizon (2007) found that teachers who achieved exceptional outcomes in mathematics for their students were more likely to have participated in ongoing mentoring, informal reflection on practice and professional learning communities.

The National Numeracy Review Report Panel (2008) cite a number of exemplary professional learning programs that have enhanced teacher practice and contributed to improved numeracy outcomes. They point out that common factors in many of these include:

- a whole school approach (primary) and faculty (secondary) to a program involving teacher commitment
- school-based and focusing on the day-to-day work of teaching
- strong school leadership
- de-privatisation of practice; teachers engaging in professional dialogue about their pedagogies, assessment practices, use of data for planning, and qualities of student work
- a focus on collaborative problem solving
- programs extended over extended period of time involving follow-up and support for further learning
- support from sources external to the school that can provide on-going input of ‘what works’ in other locations (including national and international)
- evidence-based programmes underpinned by research (2008, pp.73–74).

Timperly et al. (2007) reviewed a range of international research studies on professional learning activities that resulted in improved numeracy outcomes for students. They conclude that the following are critical factors in effective professional learning for improved numeracy outcomes.

- A supportive professional environment.
- Sufficient time and the support and guidance to make adequate use of time—ongoing professional learning requires an extended time frame of at least a year.
- Participation in some form of learning community with other colleagues at the school or with colleagues from other schools.
- Initial and ongoing support from an expert.
- The content of the professional learning which provided:
  - a range of knowledge, understandings and skills that related directly to mathematics
  - understandings of the theoretical basis for the practices being promoted, as well as the complex relationship between the key elements of discipline knowledge, pedagogy, assessment, and how students learn
  - learning about how students come to understand mathematical ideas, student reasoning, analysis, and problem solving.
- Assessment provided the impetus for challenging existing practice, the basis for instructional decisions, and the motivation to sustain professional learning as teachers saw evidence of growth in students’ learning and improved outcomes (Timperly et al., 2007, pp.92–93).

Timperly et al. (2007, p.93) conclude their discussion of professional learning for mathematics teachers by pointing out that the learning content should be a key focus in all professional learning programs for mathematics. This is in response to their finding that the quality of learning content is by far and away the most influential factor in professional learning that contributes to improved student outcomes in mathematics.

**Conclusion**

The effective teaching of numeracy knowledge and skills requires extensive knowledge of mathematical content and pedagogical content knowledge. It also requires a strong belief that all students have the potential to be numerate. Teachers need a wide repertoire of practices which they combine, integrate and adapt in different ways and combinations to suit the learning tasks and contexts and to meet individual students’ learning needs. The ability to do this well requires an ongoing commitment to analysing student learning achievement and reflecting on teaching practice and using this information to inform future professional learning and/or instruction.
References


